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Neal–Smith Criteria-Based H_∞-Modeling Technique for Assessing Pilot Rating

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The Neal–Smith (Neal, T. P., and Smith, R. E., "An In-Flight Investigation to Develop Control System Design Criteria for Fighter Airplanes," Air Force Flight Dynamics Lab., AFFDL-TR-70-74, Vols. 1 and 2, 1970) criteria for assessing handling qualities of highly augmented aircraft are rewritten in terms of H_∞-norms to arrive at a mixed-sensitivity performance index. The pitch attitude control is formulated as a mixed-sensitivity problem, which is solved by the model matching technique in the framework of the H_∞-control theory to yield H_∞-optimal pilot models. It is shown that the obtained H_∞-optimal complementary sensitivity function meets the Neal–Smith criteria so perfectly that the task difficulty is exhibited solely by the H_∞-optimal pilot models. Surveying all of the flight configurations of the Neal–Smith flight test, a method using the maximum gain gradient and the phase at a particular frequency of the H_∞-optimal pilot model is proposed to correlate pilot ratings with the pilot compensation efforts.

Introduction

HANDLING qualities specifications for highly augmented aircraft can no longer be based on their basic dynamics, but rely heavily on closed-loop analyses of pilot–aircraft systems. Attempts at correlating pilot ratings with the quantities resulting from the closed-loop analyses have yielded important relationships between pilot ratings and pilot compensation efforts, culminating in the well-known Neal–Smith criteria.[1] The criteria state that pilot rating is primarily a function of the pilot's compensation required to achieve good low-frequency performance and the closed-loop oscillatory tendencies that result. In the original work by Neal and Smith,[1] the pilot compensation required to meet the criteria is found on the Nichols chart by adjusting the gain and the lead-lag time constants of a simple pilot transfer function model. Thus, the determination of the pilot-related parameters depends on the researcher's graphical skill and insight into the system. Despite all of the efforts to enhance usefulness of Neal and Smith's work,[2–4] the works based on assumed pilot transfer functions have incurred difficulties in finding the pilot compensation uniquely.

One way to cope with the difficulties is to employ the optimal control approach to modeling the pilot loop closures.[5] In this approach, a stochastic optimal control problem with a performance index of the quadratic form is solved to yield an optimal control model (OCM) of the pilot including a Kalman filter as the state estimator. The OCM in turn may lead to the unique determination of the pilot compensation. It has been shown that with proper modeling the magnitude of the model's cost function correlates with subjective rating in a variety of tasks,[6] and the optimal pilot compensation and the optimal closed-loop parameters may duplicate Neal and Smith's results.[7] Although past works try to reflect the Neal–Smith criteria in their resulting OCMs, the performance index itself is not a manifestation of the criteria. Furthermore, the robustness implied by the criteria is not well addressed in the conventional OCM approach. Because the Neal–Smith criteria describe the standard of performance that the pilot is trying to achieve for a required tracking task, a performance index can be formed out of the criteria.

Drawing on these past works, an attempt was made at rewriting the Neal–Smith criteria in terms of H_∞ norms to yield a mixed-sensitivity problem.[8] The problem was solved in the framework of the H_∞-control theory to obtain H_∞-optimal pilot models. It is shown in Ref. 8 that the phase compensation at the bandwidth frequency of the H_∞-optimal pilot model correlates well with pilot ratings for the configurations in the Neal–Smith flight test with large short-period mode damping ratios, but it does not for the configurations with small short-period mode damping ratios.

This paper's objective is, therefore, to pursue the pilot compensation indices that correlate better with pilot ratings. The H_∞-modeling approach based on the Neal–Smith criteria is first summarized. As a result of the H_∞-optimal pilot modeling survey of the Neal–Smith flight test, a pair of pilot compensation parameters is then proposed as the indices capable of clearly dividing pilot ratings into three levels.

Neal–Smith Criteria and an H_∞-Performance Index

Figure 1 shows the pitch attitude tracking feedback system treated by Neal and Smith. The pilot transfer function is denoted by $Y_p(s)$ and the aircraft transfer function by $H(s)$, which is the pitch attitude response θ to an elevator stick force input Fs, including a flight control system (FCS). Define the following closed-loop transfer functions.

Sensitivity function:

$$\frac{\theta_e}{\theta_c} = S = \frac{1}{1 + Y_p(s)H(s)} \tag{1}$$

Complementary sensitivity function:

$$\frac{\theta}{\theta_c} = T = \frac{Y_p(s)H(s)}{1 + Y_p(s)H(s)} \tag{2}$$

where s is the Laplace transform parameter. Using T, the Neal–Smith criteria can be itemized as follows.

1) Bandwidth (BW) ω_b

$$\begin{aligned} &\geq 3.0\ \text{rad/s} \quad \text{for} \quad V_{\text{ind}} = 250\ \text{kt} \\ &\geq 3.5\ \text{rad/s} \quad \text{for} \quad V_{\text{ind}} = 350\ \text{kt} \end{aligned} \tag{3}$$

where V_{ind} denotes the trimmed indicated airspeed and BW is the frequency at which the phase angle of $T = -90$ deg.

2) Minimize the low-frequency droop of T, or desirably

$$|T| \geq -3\ \text{dB} \quad \text{for} \quad \omega \leq \omega_b \tag{4}$$

3) Minimize the resonant peak in the high-frequency range, or

$$\text{minimize } |T|_{\max} \quad \text{for} \quad \omega \geq \omega_b \tag{5}$$

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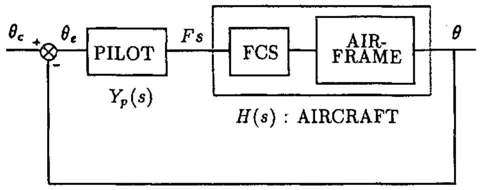

Fig. 1 Pitch attitude tracking feedback system.

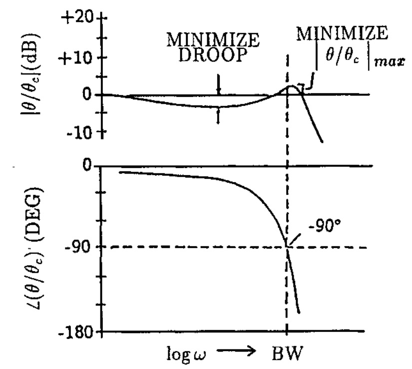

Fig. 2 Neal–Smith criteria.

These criteria are shown in Fig. 2. The low-frequency requirement for T, inequality (4), is concerned with the tracking performance. If the performance is interpreted as the mean square value of θ_e of Fig. 1, i.e., $\overline{\theta_e^2}$, to the mean square value of a random input θ_c, i.e., $\overline{\theta_c^2}$, the performance is limited by $\sup_\omega |S(j\omega)|^2$ (Ref. 9).

Therefore, criterion 2 is replaced by the criterion

$$\text{minimize } |S|_{\max} \qquad \text{for} \qquad \omega \leq \omega_b \tag{6}$$

Because criterion (6) does not guarantee that the droop should be minimized, shaping of $|T|$ is taken into consideration through the related weighting function as described later.

Putting these requirements (3), (5), and (6) together, a performance index can be proposed to represent the Neal–Smith criteria as

$$\lambda^2 = \inf_{\text{stabilizing } Y_p(j\omega)} \| |V(j\omega)S(j\omega)|^2 + |W(j\omega)T(j\omega)|^2 \|_\infty \tag{7}$$

where λ is the optimal value of the performance index and

$$\|\bullet(j\omega)\|_\infty = \sup_\omega |\bullet(j\omega)|$$

$V(j\omega)$ and $W(j\omega)$ are suitably chosen weighting functions with the roles of shaping S and T and separating a low-frequency range from a high-frequency range. The problem to find an optimal controller in the sense of Eq. (7) is a mixed-sensitivity problem to be solved in the framework of the H_∞-control theory.

Weighting Functions

Because the weighting functions $V(j\omega)$ and $W(j\omega)$ have to express the implications of the Neal–Smith criteria, they must be chosen carefully. Explicitly, they are required to shape S and T in such a way that

$$|S(j\omega)| \leq |V^{-1}(j\omega)| \leq \bar{D} \qquad \text{for} \qquad \omega \leq \omega_b \tag{8}$$

$$|T(j\omega)| \leq |W^{-1}(j\omega)| \qquad \text{for} \qquad \omega \geq \omega_b \tag{9}$$

In inequality (8), the upper bound $\bar{D}$ is required to be as small as possible. However, this condition has turned out to be too stringent for the numerical work to follow. $\bar{D}$ is set here equal to $\sqrt{2}$ by a trial-and-error estimation so that the resulting pilot model has a reasonable gain level.[10] Minimizing the droop is taken care of by shaping T through the weighting function W. The requirement (8) for $V(j\omega)$, the requirements (5) and (9) for $W(j\omega)$, and the requirement that $V(j\omega)$ and $W(j\omega)$ must be capable of distinguishing a low-frequency range from a high-frequency range with the boundary frequency of ω_b can specify the conditions for $V(j\omega)$ and $W(j\omega)$ as follows.

Table 1 Parameter values of the weighting functions

BW,		$V(s)$		$W(s)$		
rad/s	Configurations	τ_{V1}	τ_{V2}	κ_W	ζ_W	ω_b
3.0	1A–5E	0.660	0.333	0.12	0.71	3.0
3.5	6A–8E	0.571	0.286	0.08	0.71	3.5

1) For $V(j\omega)$, a) the break frequency is ω_b, and b) $|V^{-1}(j\omega)| \leq \sqrt{2}$ for $\omega \leq \omega_b$; however, $|V^{-1}(j\omega)|$ can be large for $\omega > \omega_b$, and, therefore, the relative degree of $V(s) \geq 0$.

2) For $W(j\omega)$, a) $|W^{-1}(j\omega)|$ should be as small as possible beyond ω_b and should be as flat as possible below ω_b, and b) from the properness of various functions that appear in the process of applying the model matching method, to be explained later, the relative degree of $W^{-1}(s) <$ the relative degree of $H(s)$.

The choice of the weighting functions compatible with these conditions is not unique. As a result of testing several combinations of the weighting functions,[8,10,11] the following forms are selected here inasmuch as these may yield the results well satisfying the Neal–Smith criteria, and they are simple and relatively easy to handle:

$$V(s) = \frac{\tau_{V_2} s + 1}{\tau_{V_1} s} \tag{10}$$

$$W(s) = \kappa_W \left[\left(s^2 / \omega_W^2 \right) + 2(\zeta_W / \omega_W) s + 1 \right] \tag{11}$$

The parameter values can be determined in the following way.

From condition 1a,

$$\tau_{V_2} = 1/\omega_b \tag{12}$$

and from condition 1b,

$$20 \log_{10} \left| V^{-1}(j\omega_b) \right| = 3.0\,\text{dB} \tag{13}$$

Therefore,

$$\tau_{V_1} = \frac{10^{\frac{3}{20}} \sqrt{2}}{\omega_b} \tag{14}$$

From condition 2a,

$$\omega_W = \omega_b \tag{15}$$

The parameter ζ_W is selected so that $|W^{-1}(j\omega)|$ shows no peak above ω_b, and the actual value κ_W is determined so that the phase angle of $T = -90$ deg at ω_b. The parameter values actually used in the analysis are shown in Table 1 for two kinds of the bandwidth frequency, each corresponding to the respective indicated airspeeds employed in the flight test by Neal and Smith. The gain diagrams of Eqs. (10) and (11) will be shown later together with the analytical results.

Solution Technique

The model matching technique is primarily used to solve the mixed-sensitivity problem of Eq. (7). In this technique, the controlled element $H(s)$ first undergoes the coprime factorization as

$$H(s) = N(s)/M(s) \tag{16}$$

Then, using the stable, proper and real-rational solutions X and Y to the Bezout equation,

$$N(s)X(s) + M(s)Y(s) = 1 \tag{17}$$

the controller $Y_p(s)$ that achieves internal stability is given by the Youla parameterization as

$$Y_p(s) = (X + MQ)/(Y - NQ) \tag{18}$$

where Q is an unknown parameter to be determined so as to be H_∞-optimal. The use of Eqs. (16) and (18) reduces the sensitivity function and the complementary sensitivity function to

$$S(s) = M(Y - NQ) \tag{19}$$

$$T(s) = N(X + MQ) \tag{20}$$

Equations (19) and (20) are substituted into Eq. (7) to have the performance index

$$\text{minimize}\|T_1 - T_2 Q\|_\infty \tag{21}$$

where T_1 and T_2 are stable and real-rational functions made up of the plant and the weighting functions. This is a model matching problem, the solving procedure of which is well developed.[9,12]

A few points are to be noted in actually analyzing the H_∞-optimal problem. First, the condition on the model matching technique that the plant $H(s)$ be free of poles or zeros on the imaginary axis is approximately satisfied by perturbing them slightly to the left-half plane of s: the pole $s = 0$ of $H(s)$ in Fig. 1 is replaced by $s = -\varepsilon$ ($\varepsilon > 0$, small). Second, an improper solution Q to the problem, which may arise from the employment of the weighting functions of the form of Eqs. (10) and (11), is interpreted here as the human pilot's exerting higher-order phase-lead compensation. Finally, the time delay term, $\exp(-\tau s)$, inherent in the pilot transfer function $Y_p(s)$, is first regarded as part of the plant $H(s)$. After finding an H_∞-optimal controller $G(s)$ that does not include the delay term, the delay term is restored to its initial position, as implied by the following open-loop transfer function:

$$\begin{aligned} L(s) = Y_p(s)H(s) &= (G(s)e^{-\tau s})H(s) \\ &= G(s)\tilde{H}(s) \end{aligned} \tag{22}$$

where

$$\tilde{H}(s) = H(s)e^{-\tau s}$$

The actual analysis makes use of the Padé approximation instead of the transcendental function and employs the value of τ of Ref. 1 as

$$\exp(-\tau s) = \frac{1 - (\tau s/2)}{1 + (\tau s/2)} \tag{23}$$

$$\tau = 0.3\,\text{s} \tag{24}$$

Rigorous handling of the delay term is possible. In fact, Refs. 10 and 11 give such an attempt, showing that the effect of the approximation of Eq. (23) is very small in the frequency range of interest, the uppermost frequency of which is around 10 rad/s for such a manual control system as under consideration. Reference 8 includes a comparison of the results between the model matching technique and Kwakernaak's technique,[13] which also requires a rational function approximation of a transcendental term in the plant. Note also that Ref. 10 includes a comparison between experimental pilot describing functions and corresponding H_∞-pilot models.

H_∞-Pilot Models

The aircraft transfer function treated here is given by

$$H(s) = H_\Theta(s)H_F(s) \tag{25}$$

where constant-speed transfer function of the pitch attitude to stick force is

$$H_\Theta(s) = \frac{K_\theta\left(\tau_{\theta_2} s + 1\right)}{s\left(s^2/\omega_{sp}^2 + 2\zeta_{sp}s/\omega_{sp} + 1\right)} \tag{26}$$

and the FCS transfer function is

$$H_F(s) = \frac{\tau_1 s + 1}{(\tau_2 s + 1)\left(s^2/\omega_\epsilon^2 + 2\zeta_\epsilon s/\omega_\epsilon + 1\right)} \tag{27}$$

Combinations of the parameters in Eqs. (26) and (27) and the flight configuration number assigned to each combination are summarized in Table 2 together with the pilot ratings given by the evaluation pilots of the Neal–Smith flight test.[1]

Table 2 Neal–Smith flight test configuration summary

Configuration	$1/\tau_1$	$1/\tau_{\theta_2}$	$1/\tau_2$	ω_{sp}/ζ_{sp}	$\omega_\epsilon/\zeta_\epsilon$	Pilot rating
1A	0.5	1.25	2.0	2.2/0.69	63.0/0.75	2–6
1B	2.0	1.25	5.0	2.2/0.69	63.0/0.75	3–3.5
1C	2.0	1.25	5.0	2.2/0.69	16.0/0.75	2–5
1D	∞	1.25	∞	2.2/0.69	75.0/0.67	3–5
1E	∞	1.25	5.0	2.2/0.69	63.0/0.75	6
1F	∞	1.25	2.0	2.2/0.69	63.0/0.75	8
1G	∞	1.25	0.5	2.2/0.69	63.0/0.75	8.5
2A	2.0	1.25	5.0	4.9/0.7	63.0/0.75	4–4.5
2B	2.0	1.25	5.0	4.9/0.7	16.0/0.75	2.5–6
2C	5.0	1.25	12.0	4.9/0.7	63.0/0.75	3
2D	∞	1.25	∞	4.9/0.7	75.0/0.67	2.5–3
2E	∞	1.25	12.0	4.9/0.7	63.0/0.75	4
2F	∞	1.25	5.0	4.9/0.7	63.0/0.75	3
2G	∞	1.25	5.0	4.9/0.7	16.0/0.75	7
2H	∞	1.25	2.0	4.9/0.7	63.0/0.75	5–6
2I	∞	1.25	2.0	4.9/0.7	16.0/0.75	8
2J	∞	1.25	0.5	4.9/0.7	63.0/0.75	6
3A	∞	1.25	∞	9.7/0.63	75.0/0.67	4–5
3B	∞	1.25	12.0	9.7/0.63	63.0/0.75	4.5
3C	∞	1.25	5.0	9.7/0.63	63.0/0.75	3–4
3D	∞	1.25	2.0	9.7/0.63	63.0/0.75	4
3E	∞	1.25	0.5	9.7/0.63	63.0/0.75	4
4A	∞	1.25	∞	5.0/0.28	75.0/0.67	5–5.5
4B	∞	1.25	12.0	5.0/0.28	63.0/0.75	7
4C	∞	1.25	5.0	5.0/0.28	63.0/0.75	8.5
4D	∞	1.25	2.0	5.0/0.28	63.0/0.75	8–9
4E	∞	1.25	0.5	5.0/0.28	63.0/0.75	7.5
5A	∞	1.25	∞	5.1/0.18	75.0/0.67	5–7
5B	∞	1.25	12.0	5.1/0.18	63.0/0.75	7
5C	∞	1.25	5.0	5.1/0.18	63.0/0.75	7–9
5D	∞	1.25	2.0	5.1/0.18	63.0/0.75	8.5–9
5E	∞	1.25	0.5	5.1/0.18	63.0/0.75	8
6A	0.8	2.4	3.3	3.4/0.67	63.0/0.75	5–6
6B	3.3	2.4	8.0	3.4/0.67	63.0/0.75	1–4
6C	∞	2.4	∞	3.4/0.67	75.0/0.67	2.5–5
6D	∞	2.4	8.0	3.4/0.67	63.0/0.75	5.5
6E	∞	2.4	3.3	3.4/0.67	63.0/0.75	5.5–8.5
6F	∞	2.4	0.8	3.4/0.67	63.0/0.75	6–10
7A	3.3	2.4	8.0	7.3/0.73	63.0/0.75	2–5
7B	8.0	2.4	19.0	7.3/0.73	63.0/0.75	3
7C	∞	2.4	∞	7.3/0.73	75.0/0.67	1.5–4
7D	∞	2.4	19.0	7.3/0.73	63.0/0.75	5.5
7E	∞	2.4	8.0	7.3/0.73	63.0/0.75	5–6
7F	∞	2.4	3.3	7.3/0.73	63.0/0.75	3–7
7G	∞	2.4	2.0	7.3/0.73	63.0/0.75	5–6
7H	∞	2.4	0.8	7.3/0.73	63.0/0.75	5
8A	∞	2.4	∞	16.5/0.69	75.0/0.67	4–5
8B	∞	2.4	19.0	16.5/0.69	63.0/0.75	3.5
8C	∞	2.4	8.0	16.5/0.69	63.0/0.75	3–3.5
8D	∞	2.4	3.3	16.5/0.69	63.0/0.75	2–4
8E	∞	2.4	0.8	16.5/0.69	63.0/0.75	2.5–5

Using the weighting function of Eqs. (10) and (11) and two sets of their parameter values shown in Table 1, H_∞-pilot models have been computed for all of the flight configurations of Table 2. Figure 3a shows the results of T, W^{-1}, S, V^{-1}, and L for $\omega_b = 3.0$ rad/s and Fig. 3b for $\omega_b = 3.5$ rad/s. Note that the property of the aircraft transfer functions' having no right-half plane zeros makes the results of Figs. 3a and 3b dependent exclusively on the given weighting functions; the results of Figs. 3a and 3b are independent of the flight configurations. It can be seen from Figs. 3a and 3b that the Neal–Smith criteria are well satisfied and the open-loop transfer functions $L(j\omega)$ match quite well with the crossover model.[14]

Figure 4 shows typical examples of the H_∞-optimal pilot models: those for the flight configurations 1F, 2D, 3A, and 5C of Table 2. Compared with the flight configuration 2D, which enjoys almost the best pilot rating, 1F is more of the low-frequency lead type, 3A is more of the mid-frequency lag type, and 5C is rather of the second-order lead type. The second-order lead control feature of 5C has been found common to all of the configurations of series 4 and 5. It can be shown analytically that this peculiar feature is primarily caused

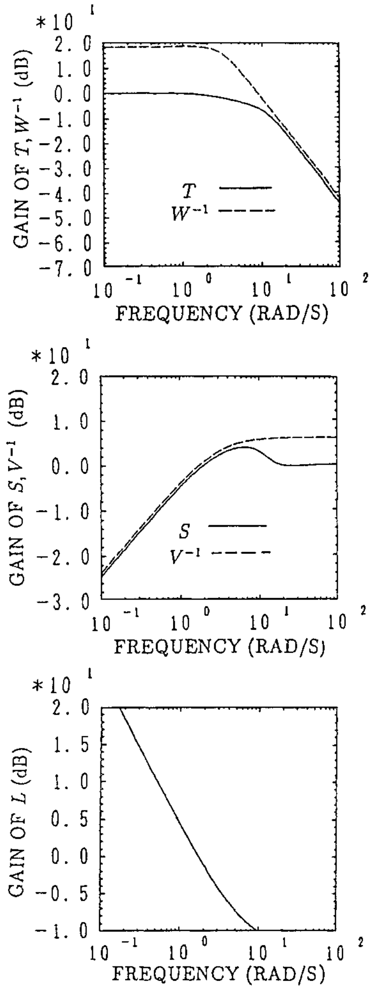

Fig. 3a **H_∞-optimal results for the configurations of BW = 3.0 rad/s.**

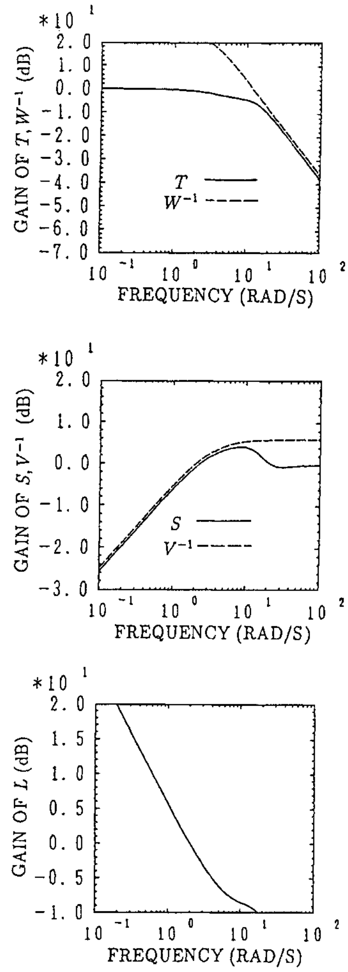

Fig. 3b **H_∞-optimal results for the configurations of BW = 3.5 rad/s.**

by the short-period mode damping ratio ζ_{sp}, where $\zeta_{sp} = 0.28$ for the series 4, and $\zeta_{sp} = 0.18$ for the series 5, whereas $\zeta_{sp} = 0.6$–0.73 for the series 1–3 and 6–8. It should be emphasized that this pilot compensation of the second-order lead type is brought about by the requirement to meet the Neal–Smith criteria of the form of Eq. (7), and it may hardly be expressed by the original Neal and Smith's simple pilot transfer function of the first-order lead-lag type.

As noticed from Fig. 4, however, the H_∞-pilot models obtained from the performance index of Eq. (7) do not show a first-order rolloff characteristic that is the high-frequency feature inherent in human pilots. This is because the performance index of Eq. (7) does not include a control-rate related term that the standard OCM approach has to account for the neuromuscular lag. It is emphasized here that in this work a pilot model is sought in a frequency range below about 10 rad/s that yields the closed-loop dynamics in the sense of Eq. (7) regardless of whether or not the model is strictly indicative of human behavior. The required compensation efforts manifested by this model will then be used as a metric to assess pilot rating. An attempt at developing an H_∞-pilot model with high fidelity is seen in Ref. 15, which includes a control-rate related term in the H_∞-performance index. It is a work for the future to combine the approach of Ref. 15 with that of this work.

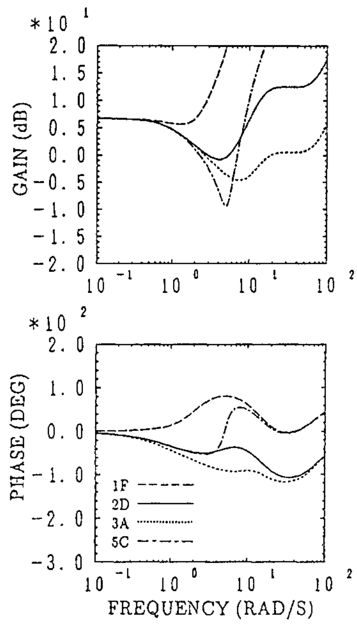

Fig. 4 **Bode diagrams of H_∞-optimal pilot models for configurations 1F, 2D, 3A, and 5C.**

Correlation with Pilot Ratings

Based on the H_∞ approach to modeling the human pilot described thus far, an attempt is made here at correlating pilot ratings with the pilot compensation efforts reflected in the H_∞-optimal pilot models. Neal and Smith[1] used the pilot's phase compensation, exclusive of the time delay effect, at the BW frequency and the magnitude of the resulting resonant peak as the measures for dividing Cooper–Harper pilot ratings into three levels: level 1, 1.0–3.5; level 2, 3.5–6.5; and level 3, 6.5–10.0. In this H_∞ approach, however, the resulting S and T satisfy the Neal–Smith criteria so well that a resonant peak does not show up. Moreover, the H_∞-optimal S and T depend only on the employed weighting functions irrespective of the flight configuration, as stated earlier. Therefore, pilot rating must be a function of only the pilot compensation manifested by the H_∞-optimal pilot model.

Following the Neal and Smith's work, it is first attempted to correlate pilot ratings with the phase compensation of the H_∞-optimal

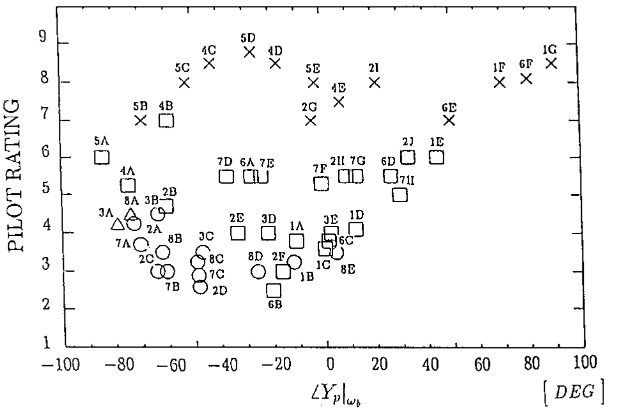

Fig. 5 Correlation between pilot ratings and the pilot's phase compensation at BW.

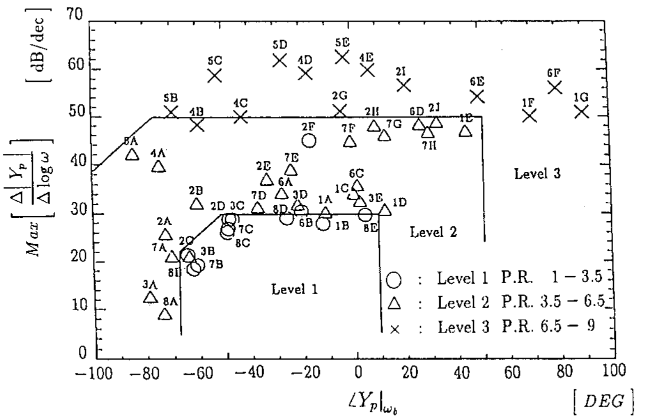

Fig. 6 Pilot ratings in the plane of the pilot's maximum gain gradient vs phase compensation at BW.

pilot model at the BW frequency including the time delay term $\angle Y_p|_{\omega_b}$. The reason for the inclusion of the time delay term is that the BW frequency for the configurations 1A–5E is not the same as that for the configurations 6A–8E, and, therefore, the phase lag due to the delay term is different between the two groups. Because the pilot compensation reflects pilot time delay in addition to the aircraft dynamics as Eq. (22) implies, it should be more indicative of the pilot's phase compensation to include the delay term. The result is given in Fig. 5, in which each pilot rating is the arithmetic mean of all of the ratings given to the particular configuration. Figure 5 shows that for the flight configurations with relatively large short-period mode damping ratios, pilot ratings change almost linearly for the worse as $\angle Y_p|_{\omega_b}$ goes farther away from the point of about −50 deg where the flight configuration 2D is situated. On the other hand, for the flight configurations with relatively small short-period mode damping ratios, 4A–5E, no such tendency can be seen. Looking back at Fig. 4, where the H_∞-optimal pilot model for 5C is shown as a typical example for the series 4 and 5, it turns out that the peculiar gain feature of the second-order lead type can be another factor showing the pilot compensation efforts. Taking into consideration that the pilot is a nonminimum phase controller, not only phase properties but also gain properties need to be used to characterize the controller. For this reason, another measure is introduced here to represent the gain properties of the H_∞-optimal pilot model: the maximum gain gradient of the H_∞-optimal pilot model within the frequency range of interest, denoted as

$$\max[\Delta|Y_p|/\Delta\log\omega],\ \text{dB/dec} \tag{28}$$

In the actual analysis for the flight configurations of Table 2, the frequency range of interest was specified as below 10 rad/s for the series 1–7 and below 16.5 rad/s ($=\omega_{sp}$) for the series 8.

Figure 6 shows the result of dividing pilot ratings into three levels with the introduction of the two measures of the pilot compensation efforts, $\angle Y_p|_{\omega_b}$ and $\max[\Delta|Y_p|/\Delta\log\omega]$. The boundaries between levels in Fig. 6 are arbitrarily chosen so that the number of flight configurations in each level is maximized.

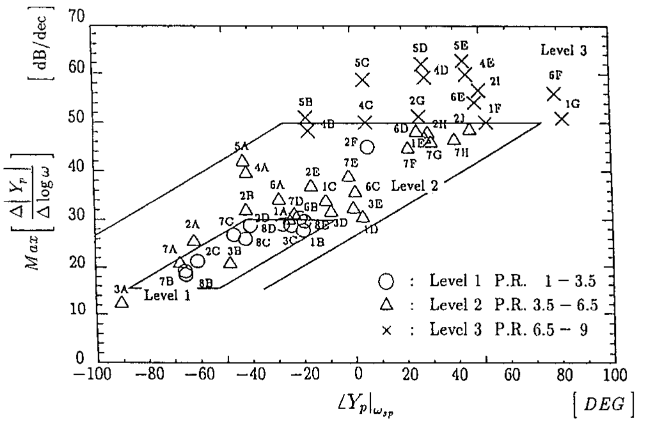

Fig. 7 Pilot ratings in the plane of the pilot's maximum gain gradient vs phase compensation at ω_{sp}.

Earlier, it was pointed out in Ref. 16 that the pilot's control action is concentrated in power on the characteristic modes of the aircraft motion. Although Ref. 16 treats lateral-directional controls of aircraft, the same can be said about longitudinal controls. For the system under consideration, the characteristic mode is the short-period mode. In fact, the frequency responses of the H_∞-optimal pilot models exhibit characteristic changes around the short-period mode natural frequency ω_{sp} as can be seen from Fig. 4. Based on this idea, pilot ratings are plotted in the plane of $\max[\Delta|Y_p|/\Delta\log\omega]$ vs $\angle Y_p|_{\omega_{sp}}$ instead of $\angle Y_p|_{\omega_b}$. The result is shown in Fig. 7, which reveals a correlation pattern similar to that of Fig. 6. To generalize this idea, however, the characteristic mode on which the pilot concentrates the control power has to be known a priori. It may depend on the flight mission and/or phase, making it necessary to ascertain the characteristic frequency by various flight experiments or simulations.

Conclusion

The Neal–Smith criteria proposed to find the pilot compensation required to achieve the standard of performance for a tracking task are rewritten in this work into a mixed-sensitivity performance index of the H_∞-control theory. The H_∞-optimal problem is solved by the model matching technique to yield H_∞-optimal pilot models for the flight test configurations of Neal and Smith.[1] The H_∞-optimal complementary sensitivity function satisfies the Neal–Smith criteria so perfectly that the task difficulty is exhibited solely by the H_∞-optimal pilot models. Surveying all of the flight configurations of the Neal–Smith flight test, a method using the maximum gain gradient and the phase at a particular frequency, the bandwidth frequency or the short-period mode natural frequency, of the H_∞-optimal pilot model is proposed to divide pilot ratings into three levels. The method is promising, and its application to other flight test results is desired to be made in the future.

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References

[1]Neal, T. P., and Smith, R. E., "An In-Flight Investigation to Develop Control System Design Criteria for Fighter Airplanes," Air Force Flight Dynamics Lab., AFFDL-TR-70-74, Vols. 1 and 2, Wright–Patterson AFB, OH, Dec. 1970.

[2]Chalk, C. R., Difranco, D. A., Lebacqz, J. V., and Neal, T. P., "Revisions to MIL-F-8785B (ASG) Proposed by Cornell Aeronautical Laboratory under Contract F33615-71-c1254," Air Force Flight Dynamics Lab., AFFDL-TR-72-41, Wright–Patterson AFB, OH, April 1972.

[3]Ashkenas, I. L., Hoh, R. H., and Craig, S. J., "Recommended Revisions to Selected Portions of MIL-F-8785B (ASG) and Background Data," Air Force Flight Dynamics Lab., AFFDL-TR-73-76, Wright–Patterson AFB, OH, Aug. 1973.

[4]Tanaka, K., "An Improved Approach to Predicting Pilot Rating Behavior," *Journal of Guidance and Control*, Vol. 4, No. 5, 1981, pp. 530–535.

[5]Kleinman, D. L., Baron, S., and Levison, W. H., "An Optimal Control Model of Human Response, Pt. I & II," *Automatica*, Vol. 6, May 1970, pp. 357–383.

[6]Hess, R. A., "Prediction of Pilot Opinion Ratings Using an Optimal Pilot Model," *Human Factors*, Vol. 19, Oct. 1977, pp. 459–475.

[7]Bacon, B. J., and Schmidt, D. K., "An Optimal Control Approach to Pilot/Vehicle Analysis and the Neal–Smith Criteria," *Journal of Guidance, Control, and Dynamics*, Vol. 6, No. 5, 1983, pp. 339–347.

[8]Goto, N., Chatani, K., and Fujita, S., "H_∞-Control Model of the Human Pilot Based on Neal–Smith Criterion," *1994 International Pacific Air and Space Technology Conference Proceedings*, Society of Automotive Engineers, Warrendale, PA, pp. 307–315; also SAE P-272, Feb. 1994.

[9]Doyle, J. C., Francis, B. A., and Tannenbaum, A. R., *Feedback Control Theory*, Macmillan, New York, 1992, Chaps. 2, 9.

[10]Goto, N., Chatani, K., and Fujita, S., "H_∞-Model of the Human Pilot Controlling Unstable Aircraft," *Proceedings of the 1995 IEEE International Conference on Systems, Man and Cybernetics* (Vancouver, BC, Canada), Inst. of Electrical and Electronics Engineers, New York, 1995, pp. 2657–2662.

[11]Goto, N., Fujita, S., and Chatani, K., "H_∞-Pilot Model for Marginally Controllable Aircraft," *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 38, No. 122, 1996, pp. 409–424.

[12]Francis, B. A., *A Course in H_∞ Control Theory*, Springer–Verlag, Berlin, 1987, Chaps. 6, 8.

[13]Kwakernaak, H., "Minimax Frequency Domain Performance and Robustness Optimization of Linear Feedback Systems," *IEEE Transactions on Automatic Control*, Vol. AC-30, No. 10, 1985, pp. 994–1004.

[14]McRuer, D. T., and Jex, H. R., "A Review of Quasi-Linear Pilot Models," *IEEE Transactions on Human Factors in Electronics*, Vol. HFE-8, No. 3, 1967, pp. 231–249.

[15]Anderson, M. R., "Standard Optimal Pilot Models," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Washington, DC, 1994, pp. 750–756; also AIAA Paper 94-3627, 1994.

[16]Goto, N., "A Statistical Method Applied to Pilot Behavior Analysis in Multiloop Systems," *Journal of Guidance and Control*, Vol. 3, No. 1, 1980, pp. 62–68.